De maculis in Sole observatis

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After Rudolf Wolf's death in 1893, his successors Alfred Wolfer (1894-1926), William Brunner (1927-1944), and Max Waldmeier (1945-1979) carried on the determination of the Zürich Sunspot Number, Rz, using the new (and differing from Wolf's) counting method introduced by Wolfer. One might have hoped that this sunspot series would be homogeneous. We shall show here that it is not.

Although the Royal Greenwich Observatory produced an unbroken series of photographs of the Sun from 1874 through 1975 that serves as the basis for the RGO-series of sunspot *areas* SA, this series was not used by the Zürich observers in construction of the sunspot number and can thus serve as an independent check on the homogeneity of Rz, on the assumption that SA is homogeneous. We shall show here that the Wolfer and Brunner series have a constant relation to SA and that they therefore can be considered to form a combined homogeneous series, but also that the Waldmeier-series calibration is higher by 17.5%.

Although to first approximation the Sunspot Areas (measured in millionths of the solar disk and corrected for foreshortening) are proportional to the sunspot number (SA ~ 15.5 Rz), the relationship is actually not linear, but is better approximated by a power law, $Rz = f(t)\ SA^p$, where the exponent p is close to 0.775. This is clearly illustrated in Figure 1 below. If both series were homogenous, the function f(t) of time t would be a constant. Inhomogeneities would result in f(t) being discontinuous with jumps at times coinciding with change of observer and/or observing and reduction procedures.

We shall concentrate on the interval (1894 through 1975) where the observing procedure (definition and counting of spots and groups) was either unchanged or intended to be so. We shall use monthly averages throughout.

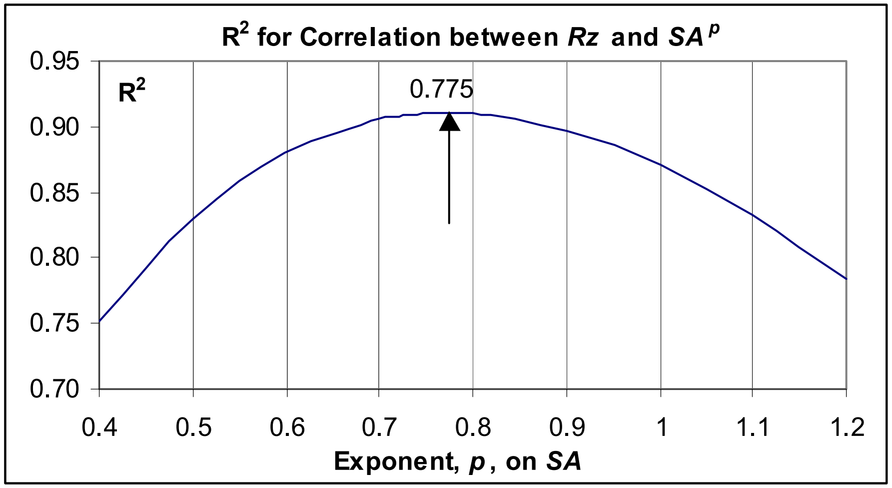

Figure 1. Coefficient of determination, R^2 (linear correlation coefficient squared), for the relation between Rz and SA^p, varying the exponent p from 0.4 to 1.2. The best correlation is obtained for p = 0.775.

The whole series from 1894 through 1975 was used. If the series were not homogeneous, additional scatter would be introduced, but for small to moderate inhomogeneities, the exponent changes but slightly.

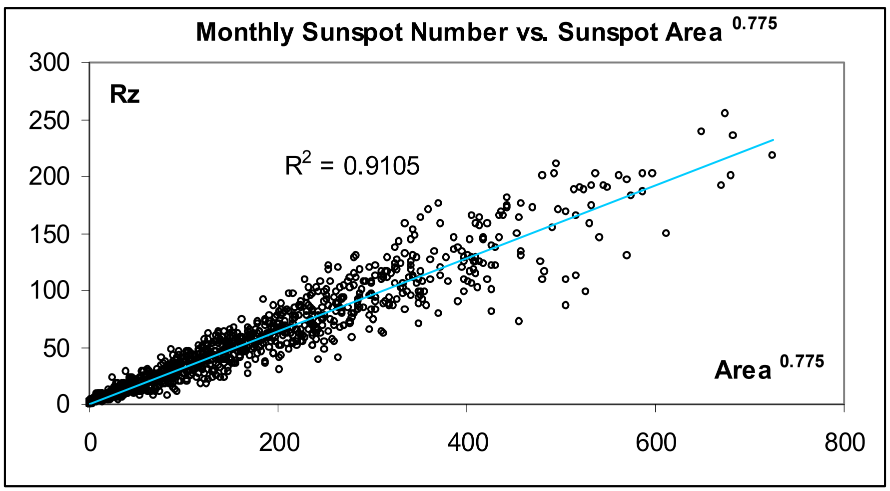

Figure 2. The (now) linear relation between monthly averages of Rz and $SA^{0.775}$ for the interval 1894-1975.

Because the relation is linear with a zero offset (it is actually 0.1314, which is so small that it can be ignored), it makes sense to form the ratio $r = SA^p/Rz$. (or equivalently its inverse). The ratio is not defined for Rz = 0; for such cases Rz is set equal to 0.01.

The ratio r, as defined in the caption to Figure 2, can now be plotted for each month as a function of time. The result is shown in Figure 3. If the series were homogeneous we would expect r to be constant across observers.

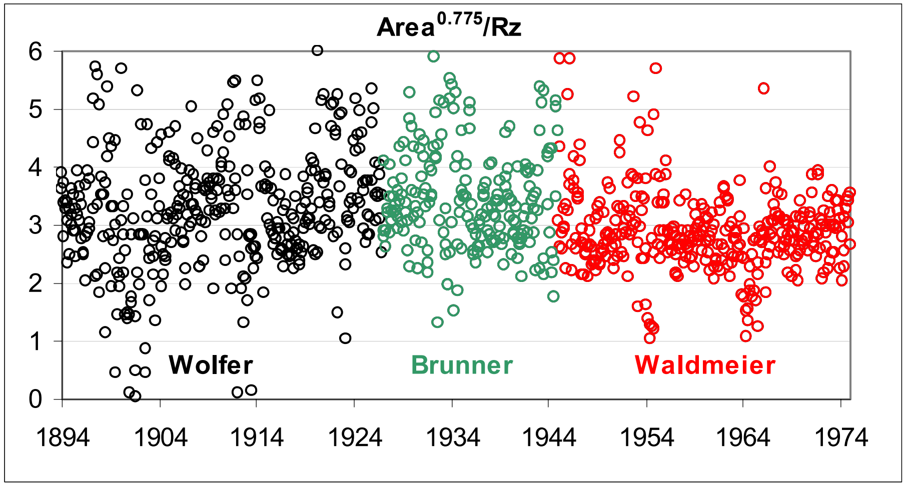

Figure 3. The ratio $r = SA^p/Rz$ for each month during 1894-1975. Each observer is marked by a color: black for Wolfer, green for Brunner, and red for Waldmeier. Since r is poorly defined for SA or Rz close to zero, the scatter at sunspot minima is larger than elsewhere. A few plotted values are cut off for $r > 6$, caused by very small values of Rz.

It is evident that while r for Wolfer and Brunner cluster around the same value, r for Waldmeier are significantly smaller. If you cannot see this outright, we can make the difference quantitative in several ways. First, we can form $\langle r \rangle = \langle SA^p \rangle / \langle Rz \rangle$ for each observer as shown in Table 1, where each average is taken over the interval of years covered by the observer:

Observer	$\langle SA^{0.775} \rangle$	<Rz>	<r>	1/<r>	Corr. Factor
Wolfer	123.71	36.50	3.3898	0.2950	1.175
Brunner	168.24	49.69	3.3859	0.2953	1.174
Waldmeier	223.45	77.46	2.8847	0.3467	1.000

The ‘correction factor’ is defined as that number by which to multiply Rz for an observer to match that of Rz by Waldmeier (*i.e.* corr. $\text{factor}_{\text{observer}} = \langle r\rangle_{\text{observer}}/\langle r\rangle_{\text{Waldmeier}}$). In other words, Rz for Wolfer-Brunner is too small by 17.5% compared to Rz observed by Waldmeier.

Second, we can plot the frequency distribution of the ratio r for each observer in Figure 4:

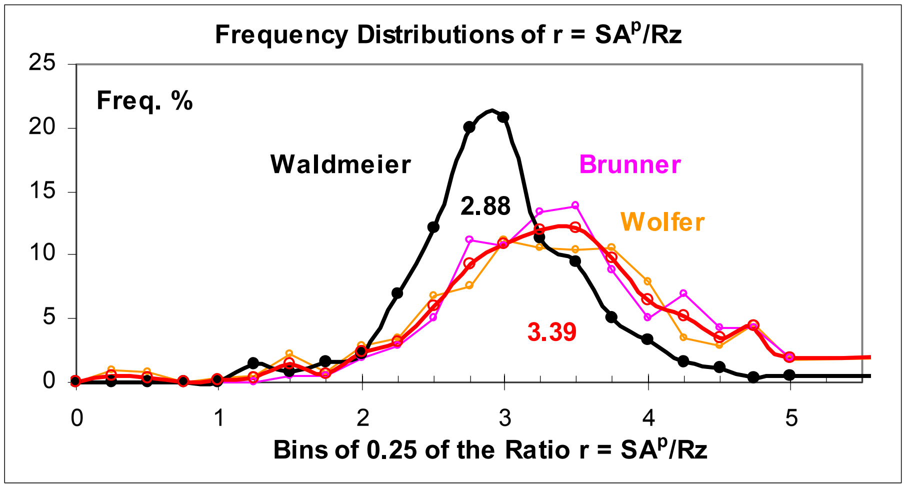

Figure 4. The frequency (in %) or the ratio r = SA^p/Rz for bins of r of 0.25. Waldmeier is shown in black; Wolfer in orange, and Brunner in pink. Since the latter two are so close, we form the average (shown in red).

The numbers show the mean values for each distribution. It is clear that the Waldmeier data form a distribution that is different from the Wolfer/Brunner data. The 17.5% difference is now evident.

Third, we can find the relation between the Rz determined by Waldmeier and SA^p, *i.e.* for the interval 1945-1975. The result is shown in Figure 5. There are a few ‘outliers’ (marked by ‘+’) which were omitted in determining the slope of the regression line. Most of these outliers are found in the first few years after the Waldmeier ‘team’ took over and may be the result of a ‘learning curve’. Friedli [2005] reports that the ‘team’ was inexperienced and that Waldmeier was worried about the calibration, and with good reason as we now know.

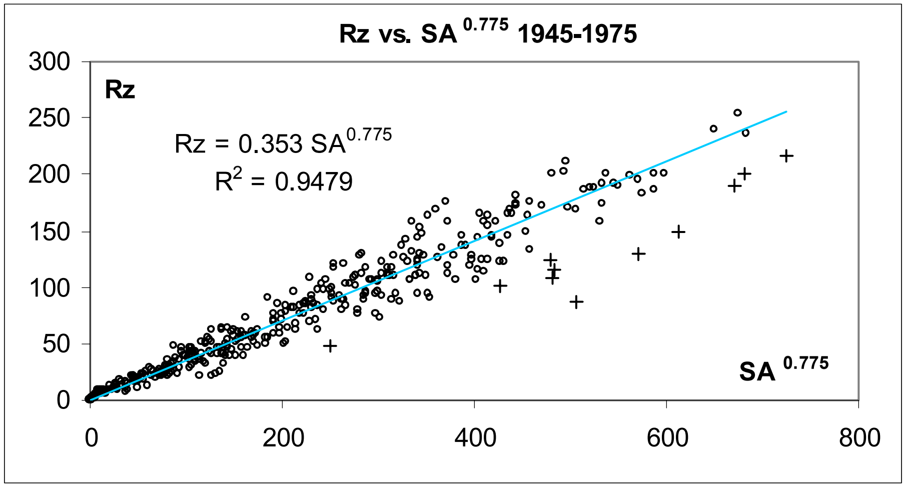

Figure 5. The linear relation between monthly averages of Rz and $SA^{0.775}$ for the interval 1945-1975, i.e. for the Waldmeier-era. A few outliers (marked with '+' symbols and probably representing a learning curve as the observers struggled to scale the rapidly increasing sunspot count during the rise of the large cycle 18) are not included in the fit:

$$Rz = 0.353\ SA^{0.775}$$

Using this relation we can calculate Rz from SA as shown in Figure 6. For the Waldmeier-era, the observed and

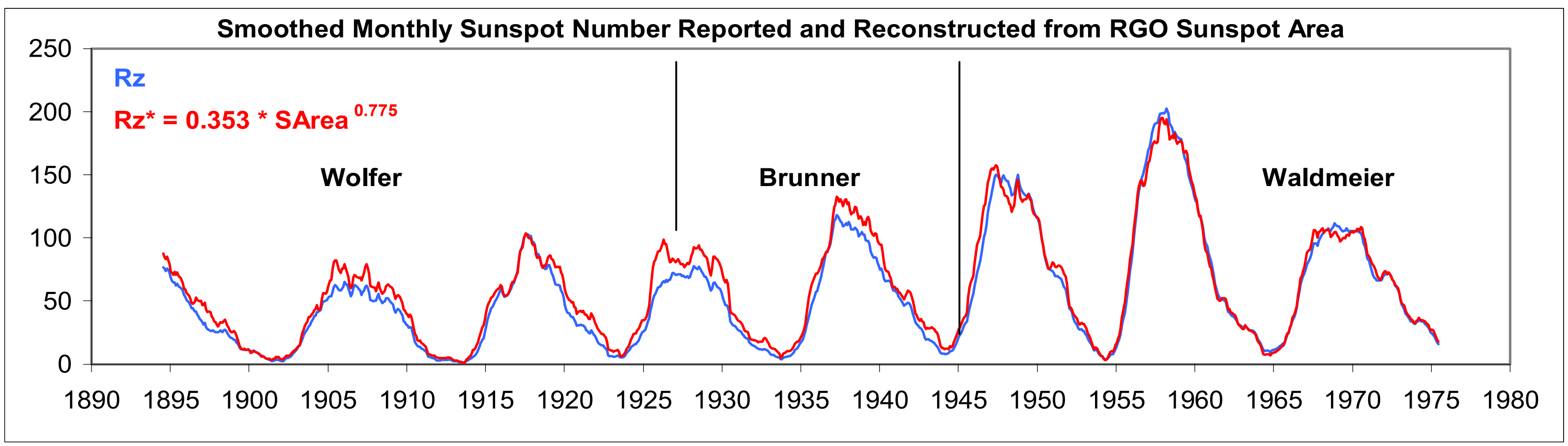

Figure 6.

calculated Rz match (by definition), but there is a clear tendency for Rz observed by Wolfer and Brunner (blue curve) to be smaller than those (red curve) determined from the photographically obtained sunspot areas using the regression equation of Figure 5.

Conclusion

From analysis of the RGO Sunspot Areas compared to the Zürich Sunspot Numbers, it emerges that there is a discontinuity of a factor of 1.175 when Waldmeier took over the production of Rz. Analysis [Svalgaard, 2007] of the geomagnetic data (daily range of East-component) yielded a factor of 1.23, in good agreement with the result obtained here from the sunspot areas. We take this as an indication that the geomagnetic calibration method is reliable (or at least gives nearly the same result as comparisons with the photographically obtained sunspot areas).

References

Friedli, T. K., Homogeneity Test of Sunspot Numbers, Dissertation, Univ. Bern, 2005.

Svalgaard, L., Calibrating the Sunspot Number Using "the Magnetic Needle", CAWSES Newsletter, *4(1)*, 2007.